MTH 2310, LINEAR ALGEBRA
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TEST 3 REVIEW

- The test will take the full period.
- You can use a calculator, but you will not need one for most questions.
- The test will cover sections 4.1-4.6 and 5.1-5.3.
- To study for the test, I recommend looking over your notes and trying to rework old problems from class, HW problems, and questions from previous quizzes. You can also work out problems from the Supplementary Exercises at the end of Chapters 4 and 5. In particular:
  (i) Chapter 4 Supplementary: # 1 (a-s), 2-11.
  (ii) Chapter 5 Supplementary: # 1 (a-r), 2, 3, and 6(a)

  The answers to most of those are in the back of the textbook if you want to check your work. You can also look over the old tests on my website. The questions that match are: Test 2: #1, 10, 11 and Test 3: all questions *except* #1(ii, iv), 7, 8
- As with the quizzes, it is important that you know not just the answer to a question, but also how to explain your answer.

Some problems to work on in class today (most of these are even-numbered problems from the textbook):

(1) True/False: If True, justify your answer with a brief explanation. If False, give a counterexample or a brief explanation.
  (a) A vector space is also a subspace.
  (b) The column space Col A is not affected by elementary row operations on A.
  (c) A linearly independent set in a subspace H is a basis for H.
  (d) Let $\mathcal{B}$ be a basis for V and $P_{\mathcal{B}}$ the change-of-coordinates matrix. Then $[\mathbf{x}]_{\mathcal{B}} = P_{\mathcal{B}}\mathbf{x}$ for all $\mathbf{x}$ in V.
  (e) The number of free variables in the equation $A\mathbf{x} = \mathbf{0}$ equals the dimension of Nul A.
  (f) If A and B are row equivalent, then their row spaces are the same.
  (g) The eigenvalues of a matrix are on its main diagonal.
  (h) A row replacement operation on a matrix A does not change the eigenvalues.
  (i) If A is invertible, then A is diagonalizable.

(2) Let W be the set of all vectors of the form $\begin{bmatrix} 4a+3b \\ 0 \\ a+b+c \\ c-2a \end{bmatrix}$. Is W a subspace of $\mathbb{R}^4$? Explain your reasoning.

(3) Consider the following two systems of equations:

$$\begin{aligned} 5x_1 + \quad x_2 - \quad 3x_3 &= 0 \\ -9x_1 + \quad 2x_2 + \quad 5x_3 &= 1 \\ 4x_1 + \quad x_2 - \quad 6x_3 &= 9 \end{aligned} \quad \text{and} \quad \begin{aligned} 5x_1 + \quad x_2 - \quad 3x_3 &= 0 \\ -9x_1 + \quad 2x_2 + \quad 5x_3 &= 5 \\ 4x_1 + \quad x_2 - \quad 6x_3 &= 45 \end{aligned}$$

It can be shown that the first system has a solution. Use this fact to explain why the second system also has a solution without making any row operations.

(4) Consider the polynomials $\mathbf{p}_1(t) = 1 + t$, $\mathbf{p}_2(t) = 1 - t$, and $\mathbf{p}_3(t) = 2$. By inspection, write a linear dependence relation among $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$. Now find a basis for Span$\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$.

(5) Are the polynomials $(1-t)^3$, $(2-3t)^2$, and $3t^2 - 4t^3$ linearly independent in $\mathbb{P}_3$? Do they form a basis?

(6) Consider the following subspace of $\mathbb{R}^4$: $\left\{ \begin{bmatrix} 3a+6b-c \\ 6a-2b-2c \\ -9a+5b+3c \\ -3a+b+c \end{bmatrix} \right\}$. Find a basis for the subspace and state its dimension.

(7) Let H be an n-dimensional subspace of an n-dimensional vector space V. Explain why $H = V$.

(8) Is it possible that all solutions of a homogeneous system of ten linear equations in twelve variables are multiples of one fixed nonzero solution? Explain your answer in terms of rank as well as the dimension of things such as the column, null, and/or row spaces.

(9) Find a basis for the eigenspace of $\begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix}$ corresponding to the eigenvalue $\lambda = -2$.

(10) Explain why A and A^T have the same characteristic polynomial (assume A is a square matrix).

(11) Diagonalize $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$.